Loss Limits and Asymmetric Volatility*

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Abstract

Leverage and volatility feedback effects have been recognized as major explanations for asymmetric volatility phenomenon. We suggest another trading-based explanation using a model of the global game. There, traders know that other traders are restricted by loss limits, but they are uncertain about the levels of other traders' limits. Facing a downward sloping demand curve, traders behave pre-emptively: traders are more willing to sell when they expect other traders sales. When bad news about the true value of the asset arrive, the pre-emptive behavior amplifies its effect and the asset price falls further. Asymmetric volatility arises because the total sales from the pre-emptive behavior will be more uncertain after a negative shock than after a positive shock. Our model supports empirical evidence in the literature that trading activity explains volatility and shows how asymmetry arises.

Keywords: Asymmetric volatility, loss limits, return skewness, financial markets, global game
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1 Introduction

Asymmetric volatility is the phenomenon that asset prices tend to experience higher variance following negative shocks. Since asymmetric volatility in stock markets was documented by Black (1976) and Christie (1982), the literature has accumulated a lot of empirical works on asymmetric volatility and has found that it is observed in various sorts of markets including bond markets (de Goeij and Marquering, 2004; Cappiello et al., 2006), currency markets (Park, 2011), emerging markets (Bekaert and Harvey, 1997) and so on.

Leverage effect, which is suggested by Black (1976) and Christie (1982), is the first explanation for the asymmetric volatility phenomenon. Although it succeeded in explaining the phenomenon to some extent, subsequent works discovered that even stocks of no-leverage firms or other assets than stock can show asymmetric volatility and alternative explanations have been pursued. Another major hypothesis, volatility feedback effect (Pindyck, 1984; French et al., 1987), is one of them. Despite that several alternative explanations have been given, it is said to be still largely unexplained (Talpsepp and Rieger, 2010).

This article contributes to the literature by suggesting another theoretical explanation which is based on trading activity. The intuition behind our explanation is as follows. In our model, traders know that other traders are restricted by loss limits, but they are uncertain about the levels of other traders' limits. Facing a downward sloping demand curve, traders behave pre-emptively: traders are more wiling to sell when they expect other traders sales. When bad news about the true value of the asset arrive, the pre-emptive behavior amplifies its effect and the asset price falls further. After a positive shock, even if a negative shock arrives in next period, the price level is high enough that no trader sells the asset for pre-emptive reason. When a negative shock hits, the situation is different: if a negative shock arrives in next period, the price level is low enough to convince some traders to pre-emptively sell the asset, while no trader behaves pre-emptively if a positive shock follows. In this way, the variance of total sales is higher when a negative shock hits than when a positive shock hits, and in turn, the variance of the asset price is higher as well.

Among others, Avramov et al. (2006) is the most related work to this article. To our best knowledge, their work is the first that suggest a trading-based explanation for the asymmetric volatility. First, they confirm that asymmetric volatility is observed at the daily frequency at which leverage effect or volatility feedback effect cannot be an explanation for asymmetric volatility. Using the method in Campbell et al. (1993), they categorize trades into informed trades and non-informational trades and examine how they affect volatility following stock price changes. Their finding is that the non-informational trades increase volatility following stock price declines, and informed trades reduce volatility following stock price increases and that is consistent with the results of Hellwig (1980) and Wang (1994). Our contribution is providing a theoretical support for the evidence in Avramov et al. (2006). Our model, focusing on loss limits, give an explanation on why trading affects volatility asymmetrically, which cannot be drawn from Hellwig (1980) or Wang

(1994). Also, while the variance of asset supply is assumed to be exogenous in their model, it can move endogenously and lead to asymmetric volatility in our model.

2 Model

Our model is a version of Morris and Shin (2004). There are three dates, indexed by $t = 0, 1, 2$. An asset is traded in $t = 1$ and is liquidated in $t = 2$. The liquidation value of the asset is given by $v_0 + v_1 + v_2$, where v_0 is a random variable that is realized in period $t = 0$, v_1 is a random variable such that

$$v_1 = \begin{cases} v_\ell & \text{with probability } 1/2, \\ v_h & \text{with probability } 1/2 \end{cases}$$

and $v_\ell < v_h$ and v_2 follows normal distribution with mean 0 and variance σ^2. We do not assume any specific distribution which v_0 follows because it is inessential. We interpret low realization of v_0 as that a bad news arrives and the asset records a negative return in $t = 0$, while high realizations are interpreted as that the asset records a positive return.[1] The value of v_1 is realized in the beginning of $t = 1$ and The value of v_2 is realized in $t = 2$.

There are two types of traders. The first type is a continuum of risk-neutral traders of measure 1. They are endowed with a unit of the asset. They participate in trading and can sell the asset in period $t = 1$ to maximize the expected value of terminal value, the value in $t = 2$. Their trading is subject to a loss limit in period 1 as described as follows. Let p denote the price of the asset in period $t = 1$, and let q_i denote the loss limit for risk-trader i. If the trader i holds the asset and $p < q_i$, then he is dismissed and get a payoff of 0.

The loss limits for risk-neutral traders randomly distribute but correlate with each other. The loss limit for i is given by

$$q_i = \theta + \eta_i$$

where θ is a common factor that imposes the correlation, η_i is an idiosyncratic factor. It is assumed that θ and η_i are independent for all i and $\{\eta_i\}_i$ is an i.i.d. sequence. Our model is abstracted from the background agency problems that motivate the loss limit, which is exogenously assumed.

The second type is a representative risk-averse trader. His preference is given by CARA utility function with the risk aversion parameter $\gamma > 0$. With the normality assumption on v_2, his asset demand d is given by

$$d = \frac{V - p}{c},$$

[1] In our model, there is no explicit trading opportunity before $t = 0$ and thus the return in $t = 0$ is not defined. It is not difficult to add a trading stage before $t = 0$ and set up a model in which the asset actually records a negative (positive) return when the realization of v is low (high).

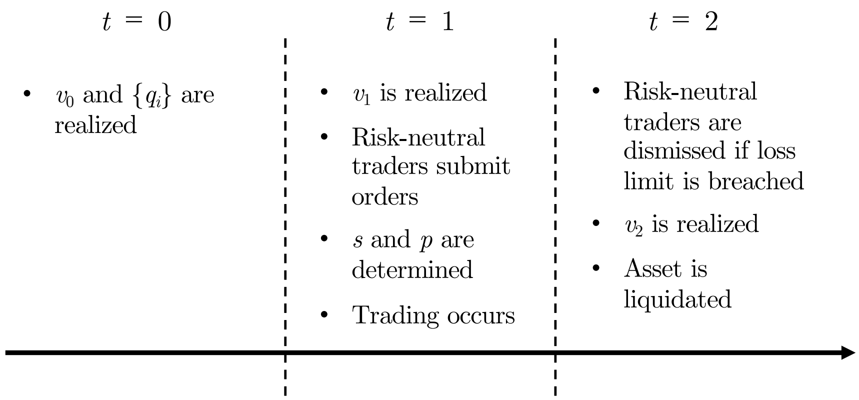

Figure 1: The timing of actions and events

where $V = v_0 + v_1$ and $c = \gamma\sigma^2$. If the total sales is s, then the asset price is

$$p = V - cs.$$

Trading occurs by matching the sales of the risk-neutral traders and limit buy orders from the representative risk-averse trader. The risk-neutral traders face uncertainty about execution price which depends on how many sales are submitted by other risk-neutral traders and where a seller's order is placed in the queue of sales for being matched with limit buy orders. We assume that a seller's place in the queue is uniformly distributed over the interval $[0, s]$ where s is the total sales. Then, the expected execution price is

$$E[p] = V - \frac{1}{2}cs.$$

The timing of actions and events is summarized in Figure 1.

The payoff to seller i for holding the asset is given by

$$u_i(s) = \begin{cases} V & \text{if } p(s) \geq q_i \\ 0 & \text{if } p(s) < q_i, \end{cases}$$

where $p(s) = V - cs$. Define $\hat{s}_i$ by $p(\hat{s}_i) = q_i$, then it is the largest amount of sales s with which trader i can execute his sales order without breaching the loss limit q_i. Using $\hat{s}_i$, the payoff for holding the asset can be rewritten as

$$u_i(s) = \begin{cases} V & \text{if } s \leq \hat{s}_i \\ 0 & \text{if } s > \hat{s}_i. \end{cases}$$

The payoff for selling the asset depends on if the execution price is higher than the loss limit.

When $s \leq \hat{s}_i$, the execution price is higher than the loss limit for sure. Therefore, the expected payoff is $v - \frac{1}{2}cs$. If $s > \hat{s}_i$, then with probability $\hat{s}_i/s$ the loss limit is not breached but with probability $1 - \hat{s}_i/s$ it is breached and the payoff is 0. Therefore, the expected payoff for selling the asset is

$$w_i(s) = \begin{cases} V - \frac{1}{2}cs & \text{if } s \leq \hat{s}_i \\ \frac{\hat{s}_i}{s}\left(V - \frac{1}{2}cs\right) & \text{if } s > \hat{s}_i. \end{cases}$$

3 Results

We concentrate on threshold equilibrium, in which the risk-neutral traders have threshold $q^*(V)$ for q_i such that if $q_i > q^*(V)$, then trader i sells the asset and holds otherwise. Morris and Shin (2004) show that the equilibrium is unique among threshold strategies and the threshold is characterized by

$$V - q^* = 2(V + q^*) \log \frac{c}{V - q^*}.$$

The equilibrium total sales $s(V)$ is given by

$$s(V) = \min \left[1, \max \left[0, \frac{\theta + \varepsilon - q^*(V)}{2\varepsilon}\right]\right].$$

We can show that $q^*(V)$ is monotone and that $s(V)$ is weakly monotone.

Lemma 1. *$s(V)$ is weakly monotone and there exists a pair $(\hat{V}_0, \hat{V}_1)$ such that*

$$s(V) = 0 \text{ if } V \geq \hat{V}_0, \tag{1}$$

$$s(V) = 1 \text{ if } V \leq \hat{V}_1, \tag{2}$$

and

$$0 < s(V) < 1 \text{ if } \hat{V}_1 < V < \hat{V}_0.$$

Our main result is Proposition 1, which shows how the initial period shock v_0 and the asset price variance $\mathrm{Var}[p]$ are related.

Proposition 1. *Let $\underline{v} = \hat{V}_1 - v_h$ and $\bar{v} = \hat{V}_0 - v_\ell$. Then, the followings hold:*

1. *When $v_0 \geq \bar{v}$,*
$$\mathrm{Var}[p] = \mathrm{Var}[v_1];$$

2. *When $\bar{v} > v_0 > \underline{v}$,*
$$\mathrm{Var}[p] > \mathrm{Var}[v_1];$$

3. *When $\underline{v} \geq v_0$,*
$$\mathrm{Var}[p] = \mathrm{Var}[v_1].$$

See Appendix for the proof.

The following example is useful for understanding the intuition behind Proposition 1.

Example 1. *We set the parameters as follows:*

$$c = 1, \varepsilon = 1, v_\ell = -0.6, v_h = 0.6,$$

and the realization of θ *is equal to 3. Then, the thresholds for* v_0 *are* $\underline{v} \approx 2.31$ *and* $\bar{v} \approx 5.54$*, and* $\mathrm{Var}[v_1] = 0.36$*. The variance of the asset price* $\mathrm{Var}[p]$ *for each level of* v_0 *is computed and drawn in the top of Figure 2. It is easy to see that when* $v_0 \leq \underline{v} \approx 2.31$ *or* $v_0 \geq \bar{v} \approx 5.54$, $\mathrm{Var}[p] = \mathrm{Var}[v_1] = 0.36$ *and that when* v_0 *is between the two thresholds* $\underline{v}$ *and* $\bar{v}$, $\mathrm{Var}[p] > \mathrm{Var}[v_1]$, *as predicted in Proposition 1.*

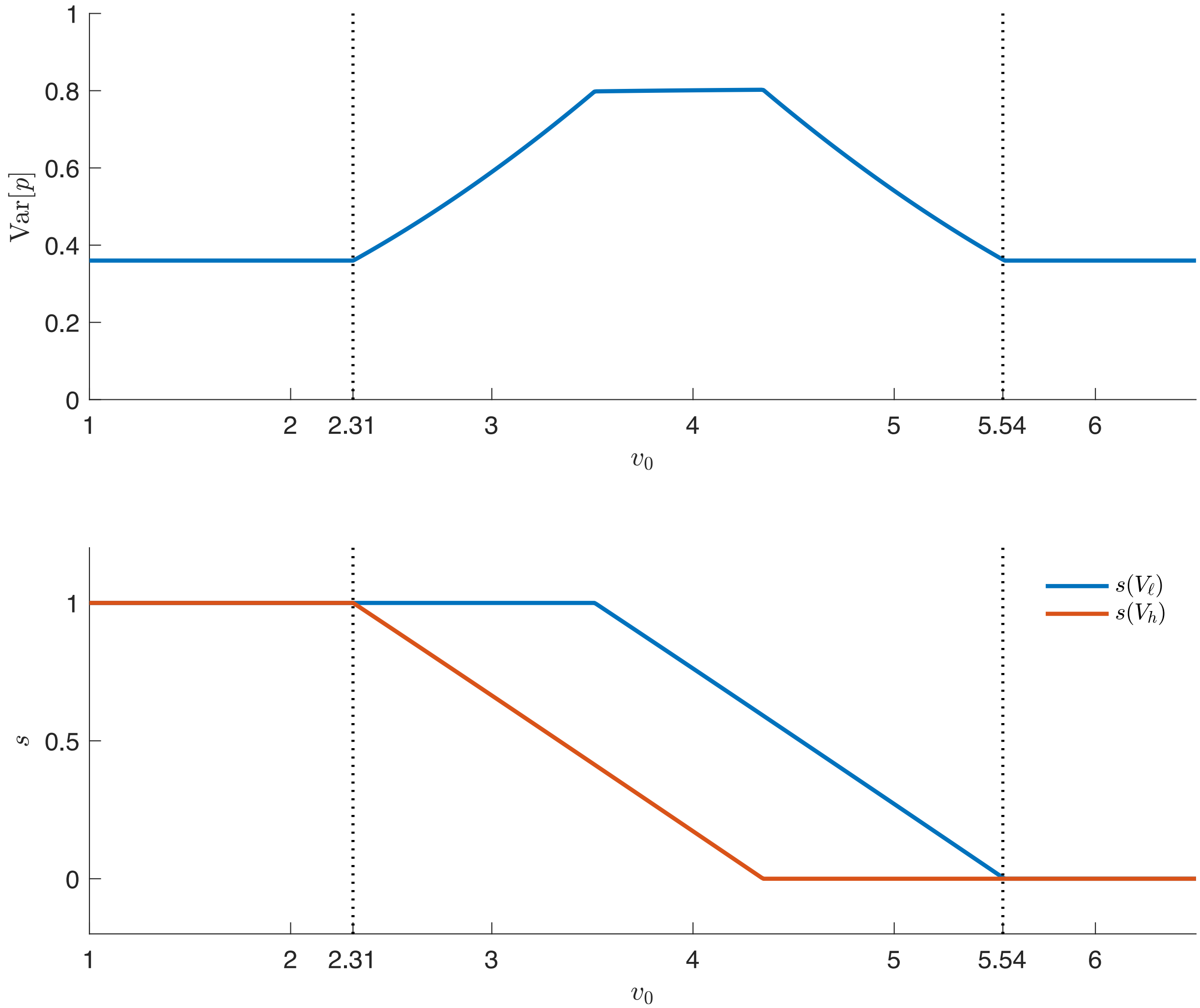

Figure 2: Variance of p and sales s for various levels of v_0

On the bottom of Figure 2, the total sales for each realization of $V = V_\ell, V_h$ is drawn. When $v_0 \leq \underline{v}$ or $\bar{v} \leq v_0$, the total sales in the two states, $s(V_\ell)$ and $s(V_h)$, are the same. When $\underline{v} < v_0 < \bar{v}$, $s(V_\ell) > s(V_h)$ holds: when a negative shock hits, pre-emptive motive leads to larger amount of sales. This uncertainty of total sales leads to the higher asset price variance in the interval.

The loss limits uniformly distribute over the interval $[2, 4]$ as $\theta = 3$ and $\varepsilon = 1$. By definition of loss limit, the initial price must be above any loss limits and thus above 4. The realization of v_0 around 6 is interpreted as "positive return" case, while the realization of v_0 around 5 is interpreted as "negative return" case. It is easy to see that the "positive return" case leads to low variance and the "negative return" case leads to high variance.

A notable feature of the relationship between the initial period shock v_0 and the asset price volatility $\mathrm{Var}[p]$ in Example 1 is that when v_0 is low, the variance $\mathrm{Var}[p]$ is low. That is because when v_0 is low, all the traders try to sell whatever the realization of V is and the variance of the total sales is 0. We do not think that this result should be taken seriously because we think that the result could be turned if some uncertainty to the asset demand is introduced.

4 Conclusion

In this note, we provided another trading-based theoretical explanation of asymmetric volatility. Our model assumes that traders are constrained by loss limits, which naturally bring asymmetry into the asset price volatility.

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A Proof

Proof of Proposition 1. Let $V_\ell = v_0 + v_\ell$ and $V_h = v_0 + v_h$.

1. Suppose that $v_0 \geq \bar{v}$. Then, $V_h > V_\ell \geq \hat{V}_0$. By Lemma 1, $s(V_\ell) = s(V_h) = 0$. Therefore,

$$\begin{aligned}
\text{Var}[p] = \text{Var}[V - cs] &= \sum_{k=\ell,h} \frac{1}{2}(V_k - cs(V_k) - E[p])^2 \\
&= \sum_{k=\ell,h} \frac{1}{2}\left(v_0 + v_k - \frac{v_0 + v_\ell + v_0 + v_h}{2}\right)^2 \\
&= \sum_{k=\ell,h} \frac{1}{2}\left(v_k - \frac{v_\ell + v_h}{2}\right)^2 \\
&= \text{Var}[v_1].
\end{aligned}$$

2. Suppose that $\bar{v} > v_0 > \underline{v}$. Then,

$$\begin{aligned}
\text{Var}[p] &= \text{Var}[V - cs] = \sum_{k=\ell,h} \frac{1}{2}(V_k - cs(V_k) - E[p])^2 \\
&= \sum_{k=\ell,h} \frac{1}{2}\left(v_0 + v_k - cs(V_k) - \frac{v_0 + v_\ell - cs(V_\ell) + v_0 + v_h - cs(V_h)}{2}\right)^2 \\
&= \frac{1}{2}\left(v_\ell - \frac{v_\ell + v_h}{2} - \frac{1}{2}[cs(V_\ell) - cs(V_h)]\right)^2 + \frac{1}{2}\left(v_h - \frac{v_\ell + v_h}{2} + \frac{1}{2}[cs(V_\ell) - cs(V_h)]\right)^2 \\
&> \sum_{k=\ell,h} \frac{1}{2}\left(v_k - \frac{v_\ell + v_h}{2}\right)^2 \\
&= \text{Var}[v_1].
\end{aligned}$$

   The inequality follows from that by Lemma 1, $cs(V_\ell) - cs(V_h) > 0$.

3. Suppose that $\underline{v} \geq v_0$. Then, $\hat{V}_1 \geq V_h > V_\ell$. By Lemma 1, $s(V_\ell) = s(V_h) = 1$. Therefore,

$$\begin{aligned}
\text{Var}[p] = \text{Var}[V - cs] &= \sum_{k=\ell,h} \frac{1}{2}(V_k - cs(V_k) - E[p])^2 \\
&= \sum_{k=\ell,h} \frac{1}{2}\left(v_0 + v_k - c - \frac{v_0 + v_\ell - c + v_0 + v_h - c}{2}\right)^2 \\
&= \sum_{k=\ell,h} \frac{1}{2}\left(v_k - \frac{v_\ell + v_h}{2}\right)^2 \\
&= \text{Var}[v_1].
\end{aligned}$$

$\square$